Welcome

Hello, and thank you for purchasing the MEAP of *Practical Probabilistic Programming*! I’m pleased to share the initial chapters of the book and look forward to interacting with you through its continued development and eventual release. This book will help you get into the new technology of probabilistic programming on the ground floor. You’ll gain an understanding of a wide variety of probabilistic modeling techniques and inference methods, and learn how to express models using the Figaro probabilistic programming system. My main goal is to make these exciting new ideas accessible to people who may not have encountered them before and help you apply it them to problems you care about.

I know that probabilistic programming is very new for most of you, so I’ll build the concepts slowly. No prior experience with probabilistic modeling is expected. I’ll also present the material in as general way as possible, teaching probabilistic programming principles that should apply to a variety of languages, while making them concrete with Figaro examples. Figaro is a library in Scala. I’m not planning to teach Scala in this book, but I will explain all the code, so you should be able to follow along even if your knowledge of Scala is limited.

What you’re looking at now is a very early section of the book, covering the basic building blocks. The first chapter provides an overview of what probabilistic programming is all about, as well as getting you started with Figaro. The next two chapters present a gentle introduction to the foundations of probabilistic modeling, with Chapter 2 telling you what a probabilistic model is and what it consists of, and Chapter 3 providing some ways to work with probabilistic models. Appendix A will help you get up and running with Scala and Figaro.

I’m hoping to provide you with Chapter 4 very shortly, which will guide you towards your first serious probabilistic program. Once you’ve got these four chapters under your belt, you will be ready for Part 2, which is all about creating probabilistic programs using a variety of modeling techniques and design patterns. Looking further ahead, Part 3 will explore different algorithms for reasoning with probabilistic programs, while Part 4 will present advanced modeling and algorithmic techniques.

I plan to bring you updates to the book on a regular basis, whether that is a new chapter or an update to an existing chapter. As you’re reading, I hope you’ll take advantage of the Author Online forum. I’ll be reading your comments and responding, and your feedback is helpful in the development process. I’m particularly interested in hearing whether the content of the book is easily understandable and if there are any points you have a hard time following. And of course, if you spot any bugs in the code or the presentation, please let me know.

Thank you,

—Avi
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Probabilistic Programming in a Nutshell

This chapter covers

- What is probabilistic programming?
- Why do I care about it? Why should my boss care?
- How does it work?
- Figaro – a system for probabilistic programming
- A comparison between writing a probabilistic application with and without probabilistic programming

In this chapter, we’ll see how we can make everyday decisions using a probabilistic model and an inference algorithm—the two main components of a probabilistic reasoning system. We’ll also see how modern probabilistic programming languages make the creation of such reasoning systems far easier than it would be in a general purpose language such as Java or Python, and introduce Figaro, the probabilistic programming language based on Scala that will be used throughout the book.

1.1 What is Probabilistic Programming?

Probabilistic programming is a way to create systems that help us make decisions in the face of uncertainty. Lots of everyday decisions involve judgment in determining relevant factors that we do not directly observe. Historically, probabilistic reasoning systems have provided a way to help make these decisions. Probabilistic reasoning combines our knowledge of a situation with the laws of probability to determine those unobserved factors that are critical to the decision. Until recently, probabilistic reasoning systems have been limited in scope, and have been hard
to apply to many real world situations. Probabilistic programming is a new approach that makes probabilistic reasoning systems easier to build and more widely applicable.

To explain probabilistic programming, we’ll start out looking at decision making under uncertainty and the judgment calls involved. Then we’ll see how probabilistic reasoning can help make these decisions. We’ll look at three specific kinds of reasoning that probabilistic reasoning systems can do. Then we’ll be able to understand probabilistic programming and how it can be used to build probabilistic reasoning systems through the power of programming languages.

1.1.1 How Do We Make Judgment Calls?
In the real world, there are rarely clear yes or no answers to the questions we care about. If we’re launching a new product, we want to know if it will sell well. We might think it will be successful, because we believe it is well designed and our market research indicates there is a need for it, but we can’t be sure. Maybe our competitor will come out with an even better product, or maybe it has some fatal flaw that will turn off the market, or maybe the economy will take a sudden turn for the worse. If we rely on being 100% sure, we will not be able to make the decision of whether or not to launch the product (Figure 1-1).
The language of probability can help make decisions like these. When launching a product, we can use prior experience with similar products to estimate the probability the product will be successful. We can then use this probability to help decide whether to go ahead in launching the product. Of course, we might not just care whether the product will be successful, but also how much revenue it will bring, or alternatively, how much we will lose if it fails. We can provide the probabilities of different outcomes to make more informed decisions.

Okay, so probabilistic thinking can help us make hard decisions and judgment calls. But how do we do that? Here’s the general principle:

**FACT** A judgment call is based on knowledge + logic

We have some knowledge of the problem we’re interested in. For example, we know a lot about our own product, and we might have done some market research to find out what customers want. We also might have some intelligence about our competitors and access to economic predictions. Meanwhile, the logic helps us get answers to our questions using the knowledge.

So, we have to have a way of specifying the knowledge, and we have to have logic for getting answers to our questions using the knowledge. Probabilistic programming is all about providing ways to specify the knowledge and logic to answer questions. Before I describe what a probabilistic programming system is, I’ll describe the more general category of probabilistic reasoning system, which provides the basic means to specify knowledge and provide logic.

### 1.1.2 Probabilistic Reasoning Systems Help Make Decisions

Probabilistic reasoning is an approach that uses a model of your domain to make decisions under uncertainty. Let’s take an example from the world of soccer. Suppose the statistics show that 9% of corner kicks result in a goal. You’re tasked with predicting the outcome of a particular corner kick. The attacking team’s center forward is 6’ 4” and known for her heading ability. The defending team’s regular goalkeeper was just carted off on a stretcher and has been replaced by a substitute playing her first game. Besides that, there’s a howling wind that makes it difficult to control long kicks. So how do you figure out the probability?

Figure 1-1 shows how you would use a probabilistic reasoning system to find the answer. You would encode your knowledge about corner kicks and all the relevant factors in a corner kick model. You would then supply evidence about this particular corner kick, namely that the center forward is tall, the goalie is inexperienced, and the wind is strong. You tell the system that you want to know whether a goal will be scored. The inference algorithm returns the answer a goal will be scored with probability 20%.
KEY DEFINITIONS

**General knowledge:** what you know to hold true of your domain in general terms, without considering the details of a particular situation

**Probabilistic model:** an encoding of general knowledge about a domain in quantitative, probabilistic terms

**Evidence:** specific information you have about a particular situation

**Query:** a property of the situation you want to know

**Inference:** the process of using a probabilistic model to answer a query given evidence

In probabilistic reasoning, you create a *model* that captures all the relevant general knowledge of your domain in quantitative, probabilistic terms. In our example, the model might be a description of a corner kick situation and all the relevant aspects of players and conditions that affect the outcome. Then, for a particular situation, you apply the model to any specific information you have to draw conclusions. This specific information is called the *evidence*. In this example, the evidence is that the center forward is tall, the goalie is inexperienced, and the wind is strong. The conclusions you draw can help you make decisions, for example, whether you should get a different goalie for the next game. The conclusions themselves are framed probabilistically, like the probability of different skill levels of the goalie.
The relationship between the model, the information you provide, and the answers to queries, is well defined mathematically by the laws of probability. The process of using the model to answer queries based on the evidence is called probabilistic inference or simply inference. Fortunately, computer algorithms have been developed that do the math for you and make all the necessary calculations automatically. These algorithms are called inference algorithms.

Figure 1-2 summarizes what we’ve just learned.

Figure 1-3: The basic components of a probabilistic reasoning system

So those, in a nutshell, are the constituents of a probabilistic reasoning system and how you interact with one. But what can you do with such a system? How does it help make decisions? The next section describes three kinds of reasoning that can be performed by a probabilistic reasoning system.

1.1.3 Probabilistic Reasoning Systems Can Reason In Three Ways

Probabilistic reasoning systems are very flexible. They can answer queries about any aspect of your situation given evidence about any other aspect. In practice, there are three kinds of reasoning that probabilistic reasoning systems do.

1. Predict future events. We’ve already seen this in Figure 1-1, where we predict whether a goal will be scored based on the current situation. Your evidence will typically consist of information about the current situation, such as the height of the center forward, the experience of the goalie, and the strength of the wind.

2. Infer the cause of events. Fast forward ten seconds. The tall center forward just scored a goal with a header, squirting under the body of the goalie. What do you think of this rookie goalkeeper, given this evidence? Can you conclude that she is poorly skilled? Figure 1-3 shows how you would use a probabilistic reasoning system to answer these questions. The model is the same corner kick model you used before to predict whether a goal would be scored. (This is a useful property of
probabilistic reasoning: the same model can be used to predict a future result as to infer what caused that result afterwards.) The evidence here is the same as before, together with the fact that a goal was scored. The query is the quality of the goalie, and the answer provides the probability of various qualities.

Figure 1-4: By altering the query and evidence, the system can now infer why a goal was scored

3. **Learn from past events to better predict future events.** Now fast forward another ten minutes. The same team has won another corner kick. Everything is similar to before in this new situation—tall center forward, inexperienced goalie—but now the wind has died down. Using probabilistic reasoning, you can use what happened in the previous kick to help you predict what will happen on the next kick. Figure 1-4 shows how you can do this. The evidence includes all evidence from last time (making a note that it was from last time), as well as the new information about the current situation. In answering the query about whether a goal will be scored this time, the inference algorithm first infers properties of the situation that led to a goal being scored the first time, such as the quality of the center forward and goalie. It then uses these updated properties to make a prediction about the new situation.
Figure 1-5: By taking into account evidence from the outcome of the last corner kick, the probabilistic reasoning system can produce a better prediction of the next corner kick.

If we think about this last kind of reasoning, we can see that this is a kind of machine learning. The system is learning from past events to better predict future events. In our example, we just learned from a single past event, but in general, we might have many past events, like a whole season’s worth of soccer games, to learn from.

PROBABILISTIC REASONING SYSTEMS AND ACCURATE PREDICTIONS Like any machine learning system, a probabilistic reasoning system will be more accurate the more data you give it. The quality of the predictions depends on two things: the degree to which the original model accurately reflects real-world situations, and the amount of data you provide. In general, the more data you provide, the less important the original model is. For example, if you’re learning from an entire soccer season, you should be able to learn the factors that contribute to a corner kick quite accurately. If you only have one game, you will need to start out with a good idea of the factors to be able to make accurate predictions about that game. In general, probabilistic reasoning systems will make good use of the given model and available data to make as accurate a prediction as possible.

All of these types of queries can help make decisions, on many levels.
- We can decide whether to substitute a defender for an attacker based on the probability a goal will be scored with or without the extra defender.
- We can decide how much to offer the goalie in her next contract negotiation based on
our assessment of her skill.

- We can decide whether to use the same goalie in the next game by using what we have learned about the goalie to help predict the outcome of the next game.

So now we know what probabilistic reasoning is. What then, is probabilistic programming?

### 1.1.4 Probabilistic Programming Systems: Probabilistic Reasoning Systems expressed in a Programming Language

Every probabilistic reasoning system uses a *representation language* to express its probabilistic models. There are a lot of representation languages out there. You may have heard of some of them, such as Bayesian networks (also known as belief networks) and hidden Markov models. The representation language controls what models can be handled by the system and what they look like. The set of models that can be represented by a language is called the *expressive power* of the language. For practical applications, we'd like to have as large an expressive power as possible.

A *probabilistic programming* system is, very simply, a probabilistic reasoning system in which the representation language is a *programming language*. When I say programming language, I mean that it has all the features you typically expect in a programming language, like variables, a rich variety of data types, control flow, functions, and so on. As we'll come to see, probabilistic programming languages are able to express an extremely wide variety of probabilistic models and go far beyond most traditional probabilistic reasoning frameworks. In other words, probabilistic programming languages have very large expressive power.

Figure 1-6 illustrates the relationship between probabilistic programming systems and probabilistic reasoning systems in general. The figure is based on Figure 1-3 and the annotations in red are taken exactly from that figure. The annotations in blue show what changes in a probabilistic programming system. The main change is that models are expressed as programs in a programming language rather than as a mathematical construct like a Bayesian network. As a result of this change, evidence, queries, and answers all apply to variables in the program. So, evidence might specify particular values for program variables, queries ask for the values of program variables, and answers are probabilities of different values of the query variables. In addition, a probabilistic programming system typically comes with a suite of inference algorithms. These algorithms apply to programs written in the language.
A probabilistic programming system is a probabilistic reasoning system that uses a programming language to represent probabilistic models. Although there are many kinds of probabilistic programming systems (see Appendix X for a survey), in this book I’m going to focus on functional, Turing-complete systems. Functional means that they are based on functional programming, but don’t let that scare you—you don’t need to know concepts like lambda functions to use functional probabilistic programming systems. All this means is that functional programming provides the theoretical foundation behind these languages that lets them represent probabilistic models. Meanwhile, Turing-complete is jargon for a programming language that can encode any computation that can be done on a digital computer. In other words, if something can be done on a digital computer, it can be done with any Turing-complete language. Most of the programming languages you are familiar with, such as C, Java, and Python, are Turing-complete. Since probabilistic programming languages are built on Turing-complete programming languages, they are extremely flexible in the types of models that can be built.

**KEY DEFINITIONS**

**Representation language:** a language for encoding your knowledge about a domain in a model

**Expressive power:** The ability of a representation language to encode various kinds of knowledge in its models

**Turing-complete:** A language that can express any computation that can be performed on a digital computer.
Probabilistic programming language: A probabilistic representation language that uses a Turing-complete programming language to represent knowledge.

Although probabilistic representation languages with programming language features have been developed that are not Turing-complete, like BUGS and Infer.NET, and these languages can be very useful, this book focuses on Turing-complete languages. From here on out, when I use the term probabilistic programming language, I am referring to a Turing-complete language.

REPRESENTING PROBABILISTIC MODELS AS PROGRAMS

But how can a programming language be a probabilistic modeling language? How can we represent probabilistic models as programs? I’m just going to hint at the answer to this question here, and save a deeper discussion until later in the book when we have a better idea of what a probabilistic program looks like.

A core idea in programming languages is execution. We execute a program to generate some output. A probabilistic program is similar, except that instead of a single execution path, there can be many execution paths, each generating a different output. The determination of which execution path is followed is specified by random choices throughout the program. Each random choice has a number of possible outcomes, and the program encodes the probability of each outcome. Therefore, a probabilistic program can be thought of as a program we randomly execute to generate an output.

Figure 1-5 illustrates this concept. In the figure, there is a probabilistic programming system that contains a corner kick program. This program describes the random process of generating the outcome of a corner kick. The program takes some inputs; in our example, these are the height of the center forward, the experience of the goalie, and the strength of the wind. Given the inputs, the program is randomly executed to generate outputs. Each random execution results in a particular output being generated. Since every random choice has multiple possible outcomes, there are many possible execution paths, resulting in different outputs. Any given output, like a goal, can be generated by multiple execution paths.
Let’s see how this program defines a probabilistic model. Any particular execution path results from a sequence of random choices having specific outcomes. Each such random choice has a probability of occurring. If we multiply all these probabilities together, we get the probability of the execution path. So, the program defines the probability of every execution path. If we imagine running the program many times, the fraction of times any given execution path will be generated is equal to its probability. The probability of an output is the fraction of times the program is run that result in that output. In Figure 1-5, a goal is generated by ¼ of the runs, so the probability of a goal is ¼.

**NOTE** You might be wondering why the block in Figure 1-7 is labeled “Random Execution” rather than “Inference Algorithm”, as it has been in other figures. Figure 1-7 shows what a probabilistic program means, as defining a random execution process, rather than how you use a probabilistic programming system, which is by using an inference algorithm to answer queries given evidence. So, although the structure of the figures is similar, they convey different concepts. As a matter of fact, random execution forms the basis for some inference algorithms as well, but there are many algorithms that are not based on simple random execution.

**MAKING DECISIONS WITH PROBABILISTIC PROGRAMMING**

It’s easy to see how you can use a probabilistic program to predict the future. Just execute the program randomly many times, using what you know about the present as inputs, and observe how many times each output is produced. In the corner kick example of Figure 1-5, we executed the program many times given the inputs of tall center forward, inexperienced goalie, and strong wind. Since ¼ of those runs resulted in a goal, we can say that the probability of a goal, given these inputs, is 25%.

The magic of probabilistic programming, however, is that it can also be used for all the kinds of probabilistic reasoning described in Section 1.1.3. Not only can it be used to predict the future, it can also be used to infer facts that led to particular outcomes. In other words,
you can “unwind” the program to discover the root causes of the outcomes. You can also apply a program in one situation, learn from the outcome, and then use what you’ve learned to make better predictions in the future. So, you can use probabilistic programming to help make all the decisions that can be informed by probabilistic thinking.

How does this work? Probabilistic programming became practical when people realized that inference algorithms that work on simpler representation languages like Bayesian networks can be extended to work on programs. In Part 3 of the book, we’ll look at a variety of inference algorithms that make this possible. Fortunately, probabilistic programming systems come with a range of built-in inference algorithms that apply automatically to your programs. All you have to do is provide your knowledge of your domain in the form of a probabilistic program, specify the evidence, and the system takes care of the inference and learning.

In this book, you’ll learn probabilistic reasoning through probabilistic programming. You’ll learn, first of all, what a probabilistic model actually is and how it can be used to draw conclusions. You’ll also learn some basic manipulations that are performed to draw those conclusions from a model made up of simple components. You’ll learn a variety of modeling techniques and how to implement them using probabilistic programming. You’ll also gain an understanding of how the probabilistic inference algorithms work, so you can design and use your models effectively. By the end of this book, you should be able to use probabilistic programming confidently to draw useful conclusions that inform your decisions in the face of uncertainty.

1.2 Why Probabilistic Programming?

Probabilistic reasoning is one of the foundational technologies of machine learning. It is used by companies such as Google, Amazon, and Microsoft to make sense of the data available to them. Probabilistic reasoning has been used for applications as diverse as predicting stock prices, recommending movies, diagnosing computers, and detecting cyber intrusions. Many of these applications use techniques you will learn in this book.

From the previous section, two things stand out:

1. Probabilistic reasoning can be used to predict the future, infer the past, and learn from the past to predict the future better;
2. Probabilistic programming is probabilistic reasoning using a Turing-complete programming language for representation.

Put these two together and we’ve got a slogan:

**FACT** Probabilistic reasoning + Turing-complete = probabilistic programming.

So, the motivation for probabilistic programming is that it takes two concepts that are very powerful in their own right and puts them together. The result is an easier and more flexible way to use computers to help make decisions under uncertainty.
1.2.1 Better Probabilistic Reasoning

Most existing probabilistic representation languages are limited in the richness of the systems they can represent. Some relatively simple languages like Bayesian networks assume a fixed set of variables and are not flexible enough to model domains where the variables themselves can change. More advanced languages with more flexibility have been developed in recent years. Some, like BUGS, also provide programming-language features like iteration and arrays, without being Turing-complete. The success of languages like BUGS shows that there is a need for richer, more structured representations. But moving to full-fledged Turing-complete languages opens a world of possibilities for probabilistic reasoning. It is now possible to model long-running processes with many interacting entities and events.

Let’s consider the soccer example again, but now imagine that we’re in the business of sports analytics and we want to recommend personnel decisions for a team. We could simply use accumulated statistics to make our decisions, but statistics don’t capture the context in which they were accumulated. We can achieve a much more fine-grained, context-aware analysis by modeling the soccer season in detail. This requires modeling many dependent events and interacting players and teams. It would be hard to imagine building this model without the data structures and control flow provided by a full programming language.

Now let’s think about the product launch example again, and look at making decisions for your business in an integrated way. The product launch is not an isolated incident, but follows phases of market analysis, research, and development, all of which have uncertainty in their outcome. The results of the product launch depend on all these phases, as well as an analysis of what else is available in the market. A full analysis will also look at how your competitors will respond to your product, as well as any new products they might bring. This problem is hard, because you have to conjecture about competing products. There may even be competitors you don’t know about yet. In this example, products are data structures produced by complex processes. Again, having a full programming language available to create the model would be helpful.

One of the nice things about probabilistic programming, however, is that if you want to use a simpler probabilistic reasoning framework, you can. Probabilistic programming systems can represent a wide range of existing frameworks, as well as systems that can’t be represented in other frameworks. This book will teach many of these frameworks using probabilistic programming. So, in learning probabilistic programming, you will also master many of the probabilistic reasoning frameworks commonly used today.

1.2.2 Better Simulation Languages

Turing-complete probabilistic modeling languages already exist. They’re commonly called simulation languages. So we know that it is possible to build simulations of complex processes like soccer seasons using programming languages. Just like probabilistic programs, these simulations are randomly executed to produce different outputs. Simulations are as widely used as probabilistic reasoning, in applications from military planning to component design to
public health to sports predictions. Indeed, the widespread use of sophisticated simulations
demonstrates the need for rich probabilistic modeling languages.

But a probabilistic program is much more than a simulation. With a simulation, you can only
do one of the things you can do with a probabilistic program: predict the future. You can’t use
it to infer the root causes of the outcomes that are observed. And, while you can update a
simulation with known current information as you go along, it’s hard to include unknown
information that must be inferred. As a result, the ability to learn from past experience to
improve future predictions and analyses is limited. In other words, you can’t really use
simulations for machine learning.

A probabilistic program is like a simulation that you can analyze, not just run. The key
insight in developing probabilistic programming is that many of the inference algorithms that
can be used for simpler modeling frameworks can also be used on simulations. Hence, we have
the ability to create a probabilistic model by writing a simulation and performing inference on
it.

One final word. Probabilistic reasoning systems have been around for a while, with
companies like Hugin, Netica, and BayesiaLab providing Bayesian network systems. However,
the much more expressive representation languages of probabilistic programming are so new
that we are just beginning to discover their powerful applications. I can’t honestly tell you that
probabilistic programming has already been used in a large number of fielded applications.
There have been some significant applications already. Microsoft has been able to determine
the true skill level of players of online games using probabilistic programming. Stuart Russell at
Berkeley has written a program to help enforce the UN Nuclear Test Ban Treaty by identifying
seismic events that could indicate a nuclear explosion. Josh Tenenbaum at MIT and Noah
Goodman at Stanford have created probabilistic programs to model human cognition with
considerable explanatory success. At Charles River Analytics, we have used probabilistic
programming to infer components of malware instances and determine their evolution. But I
believe these applications are only scratching the surface. Probabilistic programming systems
are reaching the point where they can be used by larger numbers of people to make decisions
in their own domains. By reading this book, you have a chance to get in on this new technology
at the ground floor.

1.3 Introducing Figaro: A Probabilistic Programming Language

In this book, we will be using a probabilistic programming system called Figaro. (Figaro is
named after the character from Mozart’s opera “The Marriage of Figaro”. I love Mozart and had
been playing Dr. Bartolo in the opera.) The main goal of the book is to teach the principles of
probabilistic programming, and the techniques you learn in this book should carry over to other
probabilistic programming systems. Some of the available systems are listed with a brief
description in Appendix B. A secondary goal, however, is to give you hands on experience
creating practical probabilistic programs and provide you with tools you can use right away. For
that reason, a lot of the examples will be made concrete in Figaro code.
Figaro, which is open source and maintained on GitHub, has been under development since 2009. It is implemented as a Scala library. Figure 1-8 shows how Figaro uses Scala to implement a probabilistic programming system. The figure elaborates on Figure 1-6, which describes the main components of a probabilistic programming system. We start with the probabilistic model. In Figaro, the model consists of a number of data structures known as elements. Each element represents a variable that can take on one of a number of values in your situation. These data structures are implemented in Scala, and you write a Scala program to create a model using these data structures. You can supply evidence by providing information about the values of elements, and you can specify which elements you want to know about in your query. For the inference algorithm, you choose one of Figaro’s built-in inference algorithms and apply it to your model, to answer your query given the evidence. The inference algorithms are implemented in Scala and invoking an inference algorithm is simply a Scala function call. The results of inference are probabilities of various values of your query elements.

Figure 1-8: How Figaro uses Scala to provide a probabilistic programming system

Figaro’s embedding in Scala has some major advantages. Some of these come from embedding in a general-purpose host language, compared to a standalone probabilistic language. Others come specifically because of the good properties of Scala. Here’s why it’s good to embed a probabilistic programming language in a general-purpose host language:
1. The evidence can be derived using a program in the host language. For example, you might have a program that reads a data file, processes the values in some way, and provides that as evidence for the Figaro model. It's much harder to do this in a standalone language.

2. Similarly, you can use the answers provided by Figaro in a program. For example, if you have a soccer manager's assistant program, the program can take the probability of a goal being scored to recommend to the manager what to do.

3. You can embed general-purpose code inside the probabilistic program. For example, suppose you have a physics model that simulates the trajectory of a headed ball through the air. You can incorporate this model inside a Figaro element.

4. You can use general programming techniques to build your Figaro model. For example, you might have a map containing Figaro elements corresponding to all the players in your squad and choose the appropriate elements for a situation based on the players involved in that situation.

Here are some reasons why Scala is a particularly good choice of language for embedding a probabilistic programming system in:

1. Since Scala is a functional programming language, Figaro gets to benefit from functional programming too. Functional programming has been instrumental in probabilistic programming, and many models can be written naturally in a functional manner, as I'll show in Part 2.

2. Scala is also object-oriented; in fact, one of the beauties of Scala is that it is both functional and object-oriented. Figaro is also object-oriented. As I'll describe in Part 2, object-orientation is a useful way to express several design patterns in probabilistic programming.

Finally, there are some advantages of Figaro that go beyond its embedding in Scala. These include:

1. Figaro can represent an extremely wide range of probabilistic models. The values of Figaro elements can be any type, including Booleans, Ints, Doubles, arrays, trees, graphs, and so on. The relationships between these elements can be defined by any function.

2. Figaro provides a rich framework for specifying evidence using its conditions and constraints.

3. Figaro features a good variety of inference algorithms.

4. Figaro can represent and reason about dynamic models of situations that vary over time.

5. Figaro can include explicit decisions in its models and supports inferring optimal decisions.
There are a number of reasons why Figaro is a good language for learning probabilistic
programming.

- Being implemented as a Scala library, Figaro can be used in Java and Scala programs,
making it easy to integrate into applications.

- Also related to being implemented as a library, rather than its own separate language,
you get the full functionality of the host programming language to build your models.
  Scala is an advanced, modern programming language with many useful features for
  organizing programs, and you automatically benefit from those features when using
  Figaro.

- Figaro is quite fully featured in terms of the range of algorithms it provides.

In this book, the emphasis will be on practical techniques and practical examples. Wherever
possible, I will explain the general modeling principle, as well as describing how to implement it
in Figaro. This should stand you in good stead no matter what probabilistic programming
system you end up using. Not all systems will be capable of easily implementing all the
techniques in this book. For example, there are not a lot of object-oriented probabilistic
programming systems currently. However, with the right foundation, you should find a way to
express what you need in your chosen language.

### 1.3.1 Figaro vs Java: Building A Simple Probabilistic Programming System

To illustrate the benefits of probabilistic programming and Figaro, I’m going to show you a
simple probabilistic application written two ways. First, I’ll show you how to write it in Java,
with which you might be familiar. Then I’ll show you what it looks like in Scala using Figaro.
Although Scala has some advantages over Java, that’s not the main difference I’ll point out
here. The key idea is that **Figaro provides capabilities for representing probabilistic models and
performing inference with them that are not available without probabilistic programming.**

Our little application will also serve as a “Hello world!” example for Figaro. We imagine
someone who gets up in the morning, checks if the weather is sunny, and utters a greeting
that depends on the weather. This happens two days in a row. Also, the weather on the second
day is dependent on the first day. The second day is more likely to be sunny if the first day is sunny. These English language statements can be quantified numerically by the numbers in the following tables.

**Table 1-1: Quantifying the probabilities in the “Hello world!” example**

**Today’s weather**

<table>
<thead>
<tr>
<th>Today’s weather</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunny</td>
<td>0.2</td>
</tr>
<tr>
<td>Not sunny</td>
<td>0.8</td>
</tr>
</tbody>
</table>

**Today’s greeting**

<table>
<thead>
<tr>
<th>If today’s weather is sunny</th>
<th>“Hello world!”</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Hello world!”</td>
<td></td>
<td>0.6</td>
</tr>
<tr>
<td>“Howdy, universe!”</td>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td>If today’s weather is not sunny</td>
<td>“Hello world!”</td>
<td>0.2</td>
</tr>
<tr>
<td>“Oh no, not again”</td>
<td></td>
<td>0.8</td>
</tr>
</tbody>
</table>

**Tomorrow’s weather**

<table>
<thead>
<tr>
<th>If today’s weather is sunny</th>
<th>Sunny</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Hello world!”</td>
<td></td>
<td>0.6</td>
</tr>
<tr>
<td>“Howdy, universe!”</td>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td>If today’s weather is not sunny</td>
<td>“Hello world!”</td>
<td>0.2</td>
</tr>
<tr>
<td>“Oh no, not again”</td>
<td></td>
<td>0.8</td>
</tr>
</tbody>
</table>

**Tomorrow’s greeting**

We’ll explain in the forthcoming chapters exactly how to interpret these numbers. For now, it’s enough to have an intuitive idea that today’s weather will be sunny with probability 0.2, meaning that it’s 20% likely that the weather will be sunny today. Likewise, if tomorrow’s weather is sunny, tomorrow’s greeting will be “Hello world!” with probability 0.6, meaning that
it’s 60% likely that the greeting will be “Hello world!”, while it’s 40% likely that the greeting will be “Howdy, universe!”

We’ll set for ourselves three reasoning tasks to perform with this model. We saw in Section 1.1.3 that three types of reasoning you can do with a probabilistic model are to predict the future, infer past events that led to your observations, and learn from past events to better predict the future. We’re going to do all these with our simple model. The specific tasks are:

1. Predict the greeting today.
2. Given an observation that today’s greeting is “Hello, world!”, infer whether today is sunny.
3. Learn from an observation that today’s greeting is “Hello, world!” to predict tomorrow’s greeting.

Here’s how you would do these tasks in Java.

LISTING 1-1: Hello World in Java

```java
class HelloWorldJava {
    static String greeting1 = "Hello world!";                     //A
    static String greeting2 = "Howdy, universe!";                 //A
    static String greeting3 = "Oh no, not again";                 //A

    static Double pSunnyToday = 0.2;                              //B
    static Double pNotSunnyToday = 0.8;                           //B
    static Double pSunnyTomorrowIfSunnyToday = 0.8;               //B
    static Double pSunnyTomorrowIfNotSunnyToday = 0.05;           //B
    static Double pNotSunnyTomorrowIfNotSunnyToday = 0.95;        //B
    static Double pSunnyTomorrowIfSunnyToday = 0.2;               //B
    static Double pSunnyTomorrowIfNotSunnyToday = 0.4;            //B
    static Double pGreeting1TodayIfSunnyToday = 0.6;              //B
    static Double pGreeting1TodayIfNotSunnyToday = 0.2;           //B
    static Double pGreeting3IfNotSunnyToday = 0.8;                //B
    static Double pGreeting1TomorrowIfSunnyTomorrow = 0.5;        //B
    static Double pGreeting2TomorrowIfSunnyTomorrow = 0.5;        //B
    static Double pGreeting3TomorrowIfNotSunnyTomorrow = 0.95;    //B

    static void predict() {                                       //C
        Double pGreeting1Today =                                   //C
            pSunnyToday * pGreeting1TodayIfSunnyToday +             //C
            pNotSunnyToday * pGreeting1TodayIfNotSunnyToday;        //C
        System.out.println("Today's greeting is " + greeting1 +    //C
            "with probability " + pGreeting1Today + ");           //C
    }                                                             //C

    static void infer() {                                         //D
        Double pSunnyTodayAndGreeting1Today =                     //D
            pSunnyToday * pGreeting1TodayIfSunnyToday;            //D
        Double pNotSunnyTodayAndGreeting1Today =                  //D
            pNotSunnyToday * pGreeting1TodayIfNotSunnyToday;      //D
        pSunnyTodayAndGreeting1Today /                           //D
            (pSunnyTodayAndGreeting1Today +                       //D
            pNotSunnyTodayAndGreeting1Today);                      //D
        System.out.println("If today's greeting is " + greeting1 +  //D
            ", today's weather is sunny with probability " +       //D
            pSunnyTodayGivenGreeting1Today + ");                //D
    }
}
```

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I’m not going to describe how the calculations are performed using the rules of inference here. The code uses three rules of inference: the chain rule, the total probability rule, and Bayes rule. All these rules will be explained in detail in forthcoming chapters. For now, let’s point out two major problems with this code.

1. **There’s no way to define a structure to the model.** The definition of the model is contained in a list of variable names with double values. When I described the model at the beginning of the section and showed the numbers in Table 1-1, there was a lot of structure to the model and it was relatively understandable, if only at an intuitive level.
This list of variable definitions has no structure. The meaning of the variables is buried inside the variable names, which is always a bad idea. As a result, it’s very hard to write down the model in this way, and it’s quite an error-prone process. It’s also very hard to read and understand the code afterwards and maintain it. If you need to modify the model, (e.g., the greeting also depends on whether you slept well), you will probably need to rewrite large portions of the model.

2. **Encoding the rules of inference yourself is really difficult and error-prone.** The second major problem is with the code that uses the rules of probabilistic inference to solve the problems. You have to have intimate knowledge of the rules of inference to write this code. Even if you have this knowledge, it’s very hard to write this code correctly. It’s also very hard to test whether you have the right answer. And this is an extremely simple example. For a complex application, it would be quite impractical to create reasoning code in this way.

Now let’s look at the Scala/Figaro code.

---

```scala
import com.cra.figaro.language.{Flip, Select}                      //#A
import com.cra.figaro.library.compound.If                    //#A
import com.cra.figaro.algorithm.factored.VariableElimination       //#A

object HelloWorld {
  val sunnyToday = Flip(0.2)                                       //#B
  val greetingToday = If(sunnyToday,                               //#B
    Select(0.6 -> "Hello world!", 0.4 -> "Howdy, universe!"),   //#B
    Select(0.2 -> "Hello world!", 0.8 -> "Oh no, not again"))   //#B
  val sunnyTomorrow = If(sunnyToday, Flip(0.8), Flip(0.05))        //#B
  val greetingTomorrow = If(sunnyTomorrow,                         //#B
    Select(0.6 -> "Hello world!", 0.4 -> "Howdy, universe!"),   //#B
    Select(0.2 -> "Hello world!", 0.8 -> "Oh no, not again"))   //#B

  def predict() {                                                  //#C
    val result = VariableElimination.probability(greetingToday,    //#C
      "Hello world!")                                 //#C
    println("Today's greeting is "Hello world!" +               //#C
      "with probability " + result + ".")                    //#C
  }                                                                //#C

  def infer() {                                                    //#D
    greetingToday.observe("Hello world!")                    //#D
    val result = VariableElimination.probability(sunnyToday, true) //#D
    println("If today's greeting is "Hello world!", today's " +  //#D
      "weather is sunny with probability " + result + ".")   //#D
  }                                                                //#D

  def learnAndPredict() {                                          //#E
    greetingToday.observe("Hello world!")                          //#E
    val result = VariableElimination.probability(greetingTomorrow, //#E
      "Hello world!")                                 //#E
    println("If today's greeting is "Hello world!", " +      //#E
      "tomorrow's greeting will be "Hello world!" +      //#E
      "with probability " + result + ".")                    //#E
  }                                                                //#E
}
```

---

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def main(args: Array[String]) {                                  //#F
  predict()                                                      //#F
  infer()                                                        //#F
  learnAndPredict()                                              //#F
}                                                                //#F

#A Import Figaro constructs
#B Define the model
#C Predict today's greeting using an inference algorithm
#D Infer today's weather given the observation that today's greeting is “Hello, world!” using an inference algorithm
#E Learn from observing that today's greeting is “Hello, world!” to predict tomorrow's greeting using an inference algorithm
#F Main method that performs all the tasks

I'll wait until the next chapter to explain this code in detail. For now, I want to point out that it solves the two problems with the Java code. First, the model definition describes exactly the structure of the model, in correspondence with Table 1-1. We define four variables: sunnyToday, greetingToday, sunnyTomorrow, and greetingTomorrow. Each has a definition that corresponds to Table 1. For example, here is the definition of greetingToday:

```
val greetingToday = If(sunnyToday,
  Select(0.6 -> "Hello world!", 0.4 -> "Howdy, universe!"),
  Select(0.2 -> "Hello world!", 0.8 -> "Oh no, not again"))
```

This says that if today is sunny, today's greeting is "Hello world!" with probability 0.6 and "Howdy, universe!" with probability 0.4. If today is not sunny, today's greeting is “Hello world!” with probability 0.2 and “On no, not again” with probability 0.8. This is exactly what Table 1 says for today's greeting. Since the code explicitly describes the model, it is much easier to construct, read, and maintain. And if you do need to change the model, for example by adding a sleepQuality variable, this can be done in a modular way.

Now let's look at the code to perform the reasoning tasks. It doesn't contain any calculations. Instead, it simply instantiates an algorithm (in this case, the variable elimination algorithm, one of a number of algorithms available in Figaro) and queries the algorithm to get the probability we want. Now, as I’ll describe in Part 3, this algorithm is actually based on the same rules of probabilistic inference that the Java program used. All the hard work of organizing and applying the rules of inference is taken care of by the algorithm. Even for a very large and complex model, you can simply run the algorithm and all the inference is taken care of.

1.4 Summary

In this chapter, you learned that:

- Making judgment calls requires knowledge + logic
- In probabilistic reasoning, a probabilistic model expresses the knowledge and an inference algorithm encodes the logic
• Probabilistic reasoning can be used to predict future events, infer causes of past events, and learn from past events to improve predictions
• Probabilistic programming is probabilistic reasoning where the probabilistic model is expressed using a programming language
• A probabilistic programming system uses a Turing-complete programming language to represent models and provides inference algorithms to use the models
• Figaro is a probabilistic programming system implemented in Scala that provides functional and object-oriented programming styles